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The Regulation of Fee Structures in Mutual Funds: A Theoretical Analysis

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The Regulation of Fee Structures in Mutual Funds: A Theoretical Analysis[1]

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[1]This is a preliminary draft. Comments are welcome, and may be sent to either author at the addresses given above. We are grateful to Martin Gruber for having introduced us to the topic investigated in this paper, and to Martin Gruber, Anthony Lynch, and Peter Tufano for several very helpful discussions. The second author would like to thank the National Science Foundation for support under grant SBR 94-10485.

Abstract

Existing regulations require fee structures used to compensate advisers in the mutual fund industry to be of the "fulcrum" variety, decreasing for underperforming a given index in the same way in which they increase for outperforming it. In this paper, we offer a new model for analysing the mutual fund industry, and use this model to examine the impact of restricting the fee structures that may be employed. We find little justification for existing regulations. Indeed, we find that "incentive fees" in which the adviser receives a flat fee plus a bonus for exceeding a benchmark index provide Pareto-dominant outcomes with a lower level of equilibrium volatility.

Our model also offers some insight into fee structures actually in use in the asset-management industry. We find that when leveraging is not permitted and a fulcrum fee must be employed, the equilibrium fee is a flat fee with no performance component; while if incentive fees are allowed and leveraging is permitted the equilibrium fee is an incentive fee with a large performance component. These results mesh well with observed fee structures in the mutual fund industry and the hedge fund industry, respectively.

1 Introduction

Permissible fee structures in the US mutual fund industry are laid out in the 1970 Amendment to the Investment Advisers Act of 1940. The Act, which is reviewed in Appendix A, allows mutual funds and their investment advisers to enter into performance-based compensation contracts only if the fees are of the "fulcrum" variety, that is, ones in which the adviser's fee is symmetric around a chosen index, decreasing for underperforming the index in the same way in which it increases for outperforming it. Thus, while the Act does not rule out "fraction of funds" fees in which advisers are paid a fixed percentage of the total funds under management, it does prohibit so-called "incentive fee" contracts in which advisers receive a base fee plus a bonus for exceeding a benchmark index.

The rationale offered for the prohibition of incentive fee contracts is theoretical rather than empirical in nature; that is, the ban has more to do with concerns about the inherent nature of incentive-fee contracts, rather than any actual evidence of abuse. Supporters of the prohibition, both in the SEC and in Congress, have argued that a fee structure which rewards advisers for outperforming a benchmark index without penalizing them for underperforming it provides advisers with an incentive to take excessive risk. Effectively (so the argument goes), such advisers hold an option that gives them the right to exchange a fraction of their portfolio for the benchmark portfolio. The value of this option can be increased by increasing the spread between the standard deviations of the two portfolios, leading to the concern about increased risk.

Unfortunately, this line of reasoning is far from complete. Most troubling, perhaps, is its "partial equilibrium" nature. By linking choices of risk levels solely to fee structures, it implicitly assumes that investors are passive and will not change their portfolio allocations in reaction to the altered environment. If, to the contrary, investors do choose portfolio allocations as optimal responses to fee structures and fund risk levels, it is far from obvious that admitting incentive fee structures will lead to increased levels of risk in equilibrium.

Indeed, once we move away from the partial equilibrium framework and explicitly model investor reactions, it becomes apparent that equilibrium risk levels are not the only—or even the primary—quantity with which we should be concerned. Since the objective of the legislation is to protect investors, the relevant question should be: does the prohibition of incentive fees lead to an increase in investor *welfare*? The answer is not immediate. Certainly, it is not apparent that admitting incentive fees will necessarily make the investor worse off.

This paper examines, in an equilibrium model, the extent to which the prohibition on incentive fees can be justified at a theoretical level. We find that the regulatory concern may be misplaced: we describe a set of robust models in which incentive fee structures dominate fulcrum fee structures on *all* fronts, providing Pareto-superior outcomes with lower equilibrium volatility levels. The two subsections immediately following discuss special features of the framework we employ and provide a more detailed description of our main results. A review of the related literature follows in Section 2.

1.1 Comments on Our Framework

A central question facing the modeller in constructing a useful framework for analysis of mutual funds is: who makes the decision on the choice of contract form? Following the lead of the vast literature on principal/agent problems, the majority of papers in the finance literature on mutual fund compensation have assumed that this power rests with the principal (i.e., the investor in his role as fund shareholder). Our model breaks with this tradition. We make no distinction between the fund and its investment advisers, and assign the decision on fee structure to the fund. Two considerations guide our choice in this matter.

First, while an assumption that investors choose compensation structures may be apposite in dealing with the relationship between a large client and an investment adviser, it is, perhaps, a little less suitable in the context of mutual funds. In principle, a mutual fund is controlled by its shareholders (and, indeed, is required to have "outsiders" comprise at least 40% of its board). In practice, nonetheless, the relationship between a fund and its advisers tends to be extremely close. Indeed, most management companies are responsible for establishing the funds that they advise. It appears appropriate, therefore, to explore the consequences of allowing the adviser to choose not only the risk-characteristics of the fund portfolio, but also its fee structure.

Second, our decision to have the fund's investment advisers choose the fee structure is especially appropriate from the narrow point of view of the questions motivating this paper. Indeed, if we were to adopt the standard paradigm and have the investor (in his role as principal) choose the form of the compensation contract, restricting the set of permissible contracts could end up lowering investor welfare, but certainly can never increase it. On the other hand, it makes perfect sense to ask (as we do) whether restrictions on the fund's ability to set fees can enhance investor welfare.

Our model also differs from the standard principal/agent approach in endogenizing the amount invested in the fund: investors in our model choose their portfolio allocations as optimal responses to the fund's choice of fee structure and risk levels. This appears to us to be an important consideration. Among other things, it captures the point that fee structures and risk characteristics of the fund portfolio are not chosen in isolation, but in a competitive environment for the investment dollar.[1]

[1]In a sense, therefore, our model may be thought of as a principal/agent model in which, in addition to the usual considerations, the agent also chooses the fee structure; and the principal responds by choosing the

Finally, we should mention that our results do not depend on informational asymmetries between investors and the fund. The informal arguments justifying existing restrictions on fee structures make no mention of information effects; rather, they claim that certain fee structures will inherently lead to excessive quantities of risk. It is appropriate, and maybe even desirable, that this claim be tested in a complete information setting.

1.2 Main Results

Our analysis begins in Section 3 with a commonly-used framework in this literature: that of a two-security model with one risky and one riskless asset. We characterize the equilbrium solutions in this setting first when the fund is restricted to using only fulcrum fees, then when it is restricted to using only incentive fees, and, finally, when the fund's choice set is unrestricted. A comparison of the equilibria under the first two cases reveals that incentive fees actually dominate fulcrum fees on *all* fronts in our model: (a) the utility levels of the fund and the investor are both higher under an incentive fee structure than a fulcrum fee structure; and (b) the volatility of net-of-fees returns to the investor is lower under incentive fees than under fulcrum fees. Moreover, outcomes under the incentive fee structure are, for a range of parameter values, identical to those that arise when the fund is completely unrestricted in its choice of fee structure. Thus, far from making investors worse off, allowing the fund unlimited choice in setting fee structures results in a strictly Pareto-improving situation with a lower level of volatility in net returns to the investor.

These results may appear to be counter-intuitive. In fact, they are driven primarily by the fact that the investor reacts adversely to *ceteris paribus* increases in the level of either fees or of portfolio risk. Selecting a mix that maximizes total fees under these conditions is a delicate task. Owing to their symmetric nature, fulcrum fees fail to offer enough flexibility in this direction, while incentive fees, with their asymmetric patterns enable a "better" distribution of returns between the investor and the fund. We elaborate further on this point in Section 3.5.

In Sections 4 and 5, we examine the robustness of these conclusions in two directions. Section 4 looks at the case where multiple risky assets are present. Section 5 examines the introduction of an additional dimension of moral hazard ("effort levels") in the fund's choice of actions. We find that these generalizations do no violence to our conclusions: the incentive fee structure continues to Pareto-dominate the fulcrum-fee structure, and to provide a lower level of equilibrium volatility in net returns.

Finally, although this is not the primary concern of our paper, our model also offers

amount of resources to be invested with the agent. To our knowledge, such models have not been investigated in the literature.

some insights into the fee structures commonly found in mutual funds and hedge funds. In the mutual fund industry, the overwhelming fee of choice is a flat fraction-of-funds fee with no explicit performance-adjustment component (see Appendix B for more details). In hedge funds, which are not subject to the fulcrum fee requirement and which tend to use leveraged strategies, the most popular fee structure is an incentive fee with a large performance component. These are exactly the equilibrium fee structures that arise in our model. In the absence of leveraging, we find that the equilibrium fulcrum fee is always a flat fee with no performance component. When incentive fees are allowed and leveraging is permitted, we find that the equilibrium fee is an incentive fee with a large performance component.

2 The Related Literature

This section provides a brief discussion of the theoretical literature on compensation structures in the mutual fund industry. The presentation here is meant to be indicative of the work that has been done in this area and not as a survey of the field.

Broadly speaking, there are two branches to the literature on mutual fund compensation. On the one hand are the papers that take a partial equilibrium approach and examine the reaction of managers to a *ceteris paribus* change in the fee structure. On the other hand are the papers that adopt a "full" equilibrium approach, solving for compensation structures as part of an equilibrium. Papers falling into the first group include Davanzo and Nesbit [3], Ferguson and Lestikow [4], Grinblatt and Titman [7], Grinold and Rudd [8], and Kritzman [11]. Those falling into the second group include Heinkel and Stoughton [9], Huddart [10], and Lynch and Musto [14]. Finally, there is the recent paper of Admati and Pfleiderer [1] which combines aspects of both approaches. We discuss some of these papers in more detail below.

Of the first category of papers, the most comprehensive analysis is carried out in Grinblatt and Titman [7]. Grinblatt and Titman assume that managers can risklessly capture the value of any options implicit in their payoff structure by hedging in their personal portfolios. This enables the use of results from option pricing theory in characterizing the optimal (i.e., fee maximizing) level of risk for any given contract structure. Among other things, Grinblatt and Titman show that for certain classes of portfolio strategies, adverse risk-sharing incentives are avoided when the penalties for poor performance outweigh the rewards for good performance.

Heinkel and Stoughton [9] aim to explain the predominance of fraction-of-funds fee arrangements in the asset-management industry (including, but not only, mutual funds). They employ a two-period model with heterogeneous types of managers, in which moral hazard is also present. Under some assumptions, the authors show that the optimal initial set of contracts features a smaller performance-based fee in the first period than in a first-best contract. They suggest that this reduced emphasis on the performance component in the first period is analogous to the lack of a performance-based fee in many parts of the asset-management industry.

Huddart [10] builds on the Heinkel-Stoughton model by dropping the assumption that managers are risk-neutral, and by introducing competing fund managers. He examines the problem in which the investor must decide which fund to invest in under the assumption that fees are exogeneously fixed at some proportion of assets under management. However, Huddart does show that the adoption of a performance fee can *mitigate* undesirable reputation effects and result in investors being ex-ante better off.

Lynch and Musto [14] aim to explain the fee-structures commonly found in mutual funds and hedge funds. They employ a moral hazard model in which the manager's effort is observable by the investor, but is not contractable (i.e., cannot be used as legal evidence). The manager commits to an effort level; observing this, the investor then decides on the amount of money to be invested in the fund. Lynch and Musto identify conditions in this model in which different fee structures predominate.

Our objectives differ from those of Heinkel and Stoughton [9] and Lynch and Musto [14]; it is not our primary goal to explain existing patterns of fees in the asset management industry. Our model is also different in several respects. Most importantly, the choice of fee structure in our model rests with the fund/investment adviser rather than the investor. Our model also involves the explicit use of a benchmark portfolio.[2] However, unlike Heinkel and Stoughton [9], we study a one-period model with no asymmetric information.

Admati and Pfleiderer [1] consider a scenario where the fund manager has superior information to the investor and faces a fulcrum fee structure. Their aim is to examine whether there are any conditions under which the manager would pick the investor's most desired portfolio (i.e., the portfolio that the investor would have chosen had he been possessed of the same information as the manager). There are superficial similarities between this question and that motivating our paper, but there are some fundamental differences in the analyses. First, the issue studied by Admati and Pfleiderer is the desirability of benchmarking within a fulcrum fee structure; they do not, for instance, consider incentive fee structures. We, on the other hand, take benchmarking as a given, and compare the effects of different fee structures

[2]Since mutual funds are constrained by law to using fulcrum fees, it strikes us as important in a study of mutual fund fee structures to explicitly incorporate a benchmark portfolio into the model. Without this, the notion of a fulcrum fee cannot be meaningfully defined. Moreover, since hedge funds typically use incentive fees with a substantial performance component, it appears to use that benchmarking should also play a central role in explaining *differences* in fee structures across the asset management industry.

on equilibrium payoffs. Second, Admati and Pfleiderer are not explicitly concerned with determining equilibrium fee structures and portfolio allocations. Thus, for example, they take the amount invested with the manager as exogeneous; they also compute the investor's most desired portfolio by using gross returns rather than returns net of the manager's fees. Finally, the presence of asymmetric information is central to the Admati–Pfleiderer paper, while our paper, as mentioned above, involves a symmetric information setting.

The empirical literature on the impact of different fee structures on fund performance and equilibrium risk levels is somewhat limited. Baumol, et al [2] and Lakonishok, Shleifer, and Vishny [12] have each documented the prevailing payoff structures and the extent of variation in these structures. There have also been a few direct econometric studies of the performance-fee issue, including Golec [5], Golec [6], and Lin [13]. All three of these studies find that fulcrum fees are typically used only by large (well-capitalized) firms, and, more importantly, that funds with fulcrum fees on average outperform those without such fees. However, while Golec [5] finds the performance differential to be highly significant, Lin [13] does not.

3 The Model

There are two players in our model, an investor and a fund. The investor, a representative stand-in for a large number of identical investors, has an initial wealth of $w > 0$. The investor's objective is to choose an allocation of this amount between a riskless asset and the fund so as to maximize his utility $U(\tilde{w}_T)$ from his wealth $\tilde{w}_T$ at the end of the model's single period.[3] We assume throughout that $U(\cdot)$ has a mean-variance form:

$$U(\tilde{w}_T) \;=\; E(\tilde{w}_T) - \frac{1}{2}\gamma V(\tilde{w}_T), \tag{3.1}$$

where $E(\tilde{w}_T)$ and $V(\tilde{w}_T)$ denote, respectively, the expectation and variance of $\tilde{w}_T$, and $\gamma > 0$ is a parameter signifying the investor's aversion to variance. We also assume that the amount x invested in the fund must be non-negative, i.e., that the investor cannot short the fund.

The net return on the riskless asset is normalized to zero throughout the paper. The return to the investor from the fund depends on two factors: the fee structure adopted by the fund (which determines the after-fees returns to the investor), and the return characteristics

[3]The assumption of a one-period horizon may be a limitation of our setting. However, while we remain curious about the impact of a multi-period investment horizon, we do not believe our results will be substantially altered.

of the fund portfolio. The investor takes both of these factors as given in determining his portfolio allocation.

The fees charged by the fund may depend on the realized returns r_p on the fund portfolio, as well as on the realized returns r_b on a "target" or "benchmark" portfolio. The fees, denoted $F(r_p, r_b)$, are received at the end of the period and are stated as a fraction of the asset value in the fund at that point. Thus, if the amount invested in the fund is x, and the realized returns on the two portfolios are r_p and r_b, respectively, then the terminal asset value of the fund is xr_p, so the dollar fees received by the fund are $F(r_p, r_b) \cdot xr_p$.

The return characteristics of the fund portfolio depend on the fund's action a. Let $\tilde{r}_p(a)$ denote the returns per dollar invested in the fund portfolio given a. (Throughout the paper, returns are stated in gross terms, i.e., as one plus the net return.) In this section, we shall adopt the simple and attractive setting of Grinblatt and Titman [7] and take a to denote the fraction of initial asset value invested in the benchmark portfolio, with the remainder invested at the riskless rate.[4] The set A of all actions is taken to be an interval $[0, a^{\max}]$; we allow the possibility that $a^{\max} \geq 1$.

Throughout this section, we assume that the benchmark returns $\tilde{r}_b$ follow a trinomial distribution:[5]

$$\tilde{r}_b = \begin{cases} 1+\pi_h, & \text{with probability } 1/3 \\ 1, & \text{with probability } 1/3 \\ 1-\pi_l, & \text{with probability } 1/3 \end{cases} \tag{3.2}$$

where $\pi_h, \pi_l \in (0,1)$ are constants satisfying $\pi_h > \pi_l$. We will refer to the three states as the high, middle, and low states, subscripted wherever necessary by h, m, and l, respectively. Note that the expected net return from the benchmark portfolio is $(\pi_h - \pi_l)/3$ which is strictly positive.

Observe that since the return on the riskless asset is normalized to zero, the returns $\tilde{r}_p(a)$ on the fund portfolio under the action a are given by $\tilde{r}_p(a) = a\tilde{r}_b + (1-a)$. Using (3.2), this implies

$$\tilde{r}_p(a) = \begin{cases} 1+\pi_h a, & \text{with probability } 1/3 \\ 1, & \text{with probability } 1/3 \\ 1-\pi_l a, & \text{with probability } 1/3 \end{cases} \tag{3.3}$$

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[4]The next two sections examine more complex action spaces.

[5]The popular binomial model is analytically simpler, but leads to trivial conclusions in this model: fulcrum and incentive fee equilibrium outcomes coincide along all dimensions in a binomial setting, regardless of the values of the remaining parameters.

To ensure that portfolio values remain non-negative, we must have $a^{\max} \leq 1/\pi_l$. For later analytical convenience, we make the stronger assumption that that $a^{\max} \leq 1/(2\pi_l)$.

Equilibrium

Let F_h, F_m, and F_l denote the fee charged by the fund in the three states per dollar of asset value in the fund at that state, and let $F = (F_h, F_m, F_l)$. Then, the distribution of returns to the investor, net of the fund's fees, is given by

$$R = \begin{cases} R_h &= (1-F_h)(1+\pi_h a), \quad \text{with probability } 1/3 \\ R_m &= (1-F_m), \quad \text{with probability } 1/3 \\ R_l &= (1-F_l)(1-\pi_l a), \quad \text{with probability } 1/3 \end{cases} \tag{3.4}$$

Let $E(R)$ denote the expectation of these returns, and $V(R)$ their variance. Since the riskless asset has a net return of zero, the expected terminal wealth of the investor from investing x in the fund is simply $(w - x) + xE(R)$, and the variance of this terminal wealth is $x^2 V(R)$. Thus, the investor chooses x to solve

$$\max_x \left\{ [(w - x) + xE(R)] - \frac{1}{2}\gamma x^2 V(R) \right\}.$$

The objective function is a strictly concave function of x. First-order conditions are, therefore, necessary and sufficient for a maximum, and this yields the solution[6]

$$x^* = \frac{E(R) - 1}{\gamma V(R)}. \tag{3.5}$$

Observe that x^* depends on the choice of action a taken by the fund as well as the fee structure F it adopts, through the dependence of R on these quantities. When necessary, we shall write $x^*(a, F)$ to emphasize this.

The fund is assumed to be risk-neutral.[7] Given a choice of (a, F), the expected fee EF received by the fund is given by

$$EF = \frac{1}{3}[F_h(1 + \pi_h a) + F_m + F_l(1 - \pi_l a)] \cdot x^*(a, F). \tag{3.6}$$

[6]We ignore the constraints $x \geq 0$ and $x \leq w$. The former is without loss of generality; it can never arise as an equilibrium outcome in the models we study. The latter is potentially more important. It turns out, however, that our results are qualitatively unaffected if we include this constraint; moreover, for most reasonable parameter values, the constraint is not binding in the numerical examples we develop.

[7]It appears to make no qualitative difference to the main results if the fund is risk-averse; see Appendix D.

The fund picks a and its fee structure F so as to maximize this expected fee. These optimal choices determine the equilibrium payoffs to the two players.

Admissible Fee Structures

There are two fee structures of special importance for the material that follows. The first, the so-called *fulcrum fees*, have the property that the fees per dollar invested in the fund are symmetric in the fund's performance relative to the benchmark: they increase for outperforming the benchmark in the same way that they decrease for underperforming it. We consider only linear fulcrum fees. Such fees are described by

$$F(r_p, r_b) = b_1 + b_2\, g(r_p, r_b), \tag{3.7}$$

where b_1 and b_2 are non-negative constants denoting, respectively, the base fee and the performance adjustment component, and g denotes the extent of adjustment given the actual performances. The most common form for g in practice is a simple difference of returns: $g(r_p, r_b) = (r_p - r_b)$. For analytical reasons, we use instead a *ratio* of returns:

$$g(r_p, r_b) = \left(1 - \frac{r_b}{r_p}\right). \tag{3.8}$$

This gives us closed-form solutions that, in some cases, are cleaner and easier to analyze. However, we stress that *none* of the results in this paper are qualitatively affected if we use the difference of returns form for g instead. The details are available on request from the authors.

It is almost invariably the case in practice that when fulcrum fees are used, a floor (and, by the symmetry requirement, a corresponding cap) are placed on the size of the performance adjustment component (see Appendix B). We adopt such a restriction, and require throughout the paper that the realized fees F be non-negative. This choice of zero as floor value is essentially a notational simplification. The adoption of a different floor does not appear to make an important difference to the results. The floor of zero implies, by symmetry, a cap of $2b_1$ for fees, and so our final form for fulcrum fees is:

$$F(r_p, r_b) = \begin{cases} 0, & \text{if } g(r_p, r_b) \le -\dfrac{b_1}{b_2} \\ b_1 + b_2\, g(r_p, r_b), & \text{if } -\dfrac{b_1}{b_2} < g(r_p, r_b) \le \dfrac{b_1}{b_2} \\ 2b_1, & \text{if } g(r_p, r_b) > \dfrac{b_1}{b_2} \end{cases} \tag{3.9}$$

The second class of fees of importance are *incentive fees*. Like fulcrum fees, incentive fees are described by two parameters b_1 and b_2, with b_1 denoting the base fee level, and b_2 the performance adjustment component. However, unlike fulcrum fees, the performance adjustment component must remain non-negative, and the total fee is given by

$$F = b_1 + b_2 \max\{g(r_p, r_b), 0\}, \tag{3.10}$$

where g is defined in expression (3.8) above.

In the analysis that follows, we compare equilibrium outcomes under three settings: when the fund is limited to using only fulcrum fees, when it is limited to using only incentive fees, and when it is unrestricted in its fee choices. Our decision to compare the outcomes under a fulcrum fee regime to not just the unrestricted model, but also to a restricted model in which only incentive fees are used is based on two considerations. First, the existing legislation on mutual fund fees is motivated explicitly by fear of the consequences of *incentive* fee structures. Second, from a practical standpoint, incentive fee structures are commonly used in relationships between investors and investment advisers where they are legal (e.g., in hedge funds). In contrast, as is well known, unrestricted equilibrium contracts in principal/agent models often take on unrealistically complex and unintuitive forms.

3.1 Equilibrium under Fulcrum Fees

In a fulcrum fee structure, the fund picks two parameters b_1 and b_2 denoting, respectively, its base fee and performance component. Given (b_1, b_2) and its portfolio choice a, a little algebra shows that the fees F_h, F_m, F_l received by the firm in the three states are given by

$$\begin{aligned} F_h &= \max\{0, \min\{2b_1, b_1 + b_2\pi_h(a-1)/(1+\pi_h a)\}\} \\ F_m &= b_1 \\ F_l &= \max\{0, \min\{2b_1, b_1 - b_2\pi_l(a-1)/(1-\pi_l a)\}\} \end{aligned} \tag{3.11}$$

Using this together with (3.4), the net-of-fees return distribution to the investor may be computed for any choice of (b_1, b_2, a). In principle, solving for the equilibrium is now a straightforward process. Using the distribution of net-of-fees returns, we can identify, via (3.5), the optimal x^* for the investor as a function of (a, b_1, b_2). Then, we use this optimal choice and expression (3.11) for the fees to obtain the expected fee in terms of (a, b_1, b_2). Maximizing this expected fee then yields the optimal choices of (a, b_1, b_2), and thereby the players' equilibrium payoffs.

In practice, because $a^{\max}$ is unspecified, the last step is tricky. We use, therefore, a two-step procedure. First, we hold the portfolio choice a fixed at some arbitrary level, and identify the equilibrium payoffs for the fund and the investor for this fixed a. Then, for any possible value of $a^{\max}$, we identify the value of $a \in [0, a^{\max}]$ that maximizes the fund's equilibrium payoffs.

The following proposition summarizes equilibrium outcomes when a is held fixed. It may also be some independent interest insofar as it relates equilibrium fee structures to the ability to leverage.

Proposition 3.1 *For fixed a, the equilibrium fulcrum fee structure is as follows:*

1. *If $a \leq 1$, then the equilibrium fulcrum fee contract is a flat fee contract, i.e., we have $b_1 > 0$ and $b_2 = 0$. The base fee $b_1 > 0$ is given by*

$$b_1 = \frac{a(\pi_h - \pi_l)}{6 + a(\pi_h - \pi_l)}. \tag{3.12}$$

2. *If $a > 1$, then b_1 and b_2 are both positive and are given by*

$$b_1 = \frac{a\pi_h(\pi_h - \pi_l)}{7\pi_h + 2a\pi_h^2 - \pi_l - 2a\pi_h\pi_l}. \tag{3.13}$$

$$b_2 = \frac{a\pi_h(\pi_h - \pi_l)(1 + a\pi_h)}{(a-1)(7\pi_h + 2a\pi_h^2 - \pi_l - 2a\pi_h\pi_l)}. \tag{3.14}$$

Proof See Appendix C.1. □

Closed-form expressions for the investor's equilibrium expected utility and the fund's equilibrium expected fees for a fixed value of a may be derived using Proposition 3.1. Letting $\xi = \pi_h^2 + \pi_h\pi_l + \pi_l^2$, these these expressions are given by the following. When $a \leq 1$:

$$EU = \frac{(1 + 16\gamma)\xi - 3\pi_h\pi_l}{16\gamma\xi} \tag{3.15}$$

$$EF = \frac{[3 + a(\pi_h - \pi_l)]\,(\pi_h - \pi_l)^2}{24\gamma\xi} \tag{3.16}$$

If $a > 1$, then the expressions are

$$EU = \frac{(1 + 16\gamma)\xi - 3\pi_h\pi_l}{16\gamma\xi} \tag{3.17}$$

$$EF = \frac{(\pi_h - \pi_l)^2(2a\pi_h^2 - \pi_l + 2\pi_h(2 - a\pi_l))}{24\gamma\pi_h\xi} \tag{3.18}$$

Observe that the investor's equilibrium payoffs do not depend on a in any way. Some simple calculation also reveals that the fund's equilibrium payoffs are strictly increasing in a for both $a \leq 1$ as well as for $a > 1$. Moreover, a comparison between the equilibrium expected fee when $a \leq 1$ and $a > 1$ reveals that the latter is always strictly larger. It follows immediately that

Proposition 3.2 *In a fulcrum fee regime, the optimal portfolio choice for the fund is always* $a = a^{\max}$.

1. *If $a^{\max} \leq 1$, the equilibrium fee is a flat fee with b_1 given by (3.12). The investor's expected utility level and the fund's expected fees in equilibrium are given by (3.15) and (3.16), respectively.*

2. *If $a^{\max} > 1$, the equilibrium fee structure is given by (3.13)-(3.14). The investor's expected utility level and the fund's expected fees in equilibrium are given by (3.17) and (3.18), respectively.*

Proof See Appendix C.2. □

3.2 Equilibrium under Incentive Fees

Under incentive fees, the fund picks a base fee b_1 and a performance-dependent fee b_2. Given the action a, the fee to the fund (per dollar of terminal asset value) is

$$F = b_1 + b_2 \cdot \max\{1 - \tilde{r}_b/\tilde{r}_p(a), 0\}. \tag{3.19}$$

Using the expressions (3.2) and (3.3) for the benchmark and fund portfolio returns, respectively, we can see that whether the fund portfolio outperforms the benchmark portfolio in a particular state depends on the level of a. If $a < 1$, then the fund's fees are distributed as

$$\begin{aligned} F_h &= b_1 \\ F_m &= b_1 \\ F_l &= b_1 - b_2\pi_l(a-1)/(1-\pi_l a) \end{aligned} \tag{3.20}$$

On the other hand, if $a > 1$, then the fund's fees are distributed as

$$\begin{aligned} F_h &= b_1 + b_2\pi_h(a-1)/(1+\pi_h a) \\ F_m &= b_1 \\ F_l &= b_1 \end{aligned} \tag{3.21}$$

The net-of-fees returns to the investor may be obtained from these fees using expression (3.4). We omit these expressions here. To identify the equilibrium, we proceed now as in the fulcrum fee case. We use the distribution of net-of-fees returns to identify the optimal amount invested in the fund for each (a, b_1, b_2). Using this, we solve for the optimal (a, b_1, b_2) by first identifying equilibrium payoffs for a fixed a, and then solving for the optimal a. The following result describes the equilibrium fee structure for each fixed a:

Proposition 3.3 *For fixed a, the equilibrium incentive fee structure is as follows:*

1. *When $a \leq 1$, the equilibrium incentive fee contract is a flat-fee contract with $b_1 > 0$ and $b_2 = 0$. The base fee b_1 is the same as in the fulcrum-fee case, and is given by (3.12).*

2. *When $a > 1$, the equilibrium incentive fee contract is a pure performance-fee contract, i.e., $b_1 = 0$ and $b_2 > 0$. Letting $\xi = (\pi_h^2 + \pi_h\pi_l + \pi_l^2)$ as above, b_2 is given by*

$$b_2 = \frac{a(\xi - \pi_l\sqrt{3\xi})}{(a-1)\pi_h(\pi_h + 2\pi_l)} \tag{3.22}$$

Proof See Appendix C.3. □

These fee structures may be used to solve for the equilibrium payoffs to the players for each fixed a. Since the equilibrium fee structure under incentive fees is the same as that which obtains under fulcrum fees when $a \leq 1$, equilibrium levels of expected utility and expected fees in this case are given by (3.15) and (3.16), respectively. When $a > 1$, equilibrium levels of expected utility and expected fees are given by

$$EU = \frac{\left(\xi(1+8\gamma) + 3\pi_l^2 - 2\sqrt{3\xi}(\pi_l - 2\pi_h)\right)}{4\gamma\left(2\xi + \pi_h\sqrt{3\xi}\right)} \tag{3.23}$$

$$EF = \frac{\left(\sqrt{3\xi} - 3\pi_l\right)\left(\xi - \pi_l\sqrt{3\xi}\right)}{6\gamma\pi_l\left[2\xi + \pi_h\sqrt{3\xi}\right]} \tag{3.24}$$

Observe that in this case the equilibrium payoffs to both the investor and the fund are independent of a. Comparing the expected fees for the fund when $a > 1$ to that which obtains when $a < 1$ establishes that the former is strictly larger, and therefore, that:

Proposition 3.4 *In an incentive fee regime:*

1. *When $a^{\max} \leq 1$, the optimal portfolio choice for the fund is $a^* = a^{\max}$. The equilibrium fee structure and payoffs are the same as in the corresponding fulcrum fee case, and are given by (3.12), (3.15) and (3.16), respectively.*
2. *When $a^{\max} > 1$, any action $a > 1$ is an optimal portfolio choice for the fund. The optimal fee structure is a pure performance fee with $b_1 = 0$ and b_2 given by (3.22). The equilibrium levels of expected utility and expected fees are given by (3.23) and (3.24), respectively.*

Proof See Appendix C.4. □

3.3 Comparison of the Equilibrium Outcomes

Having identified the equilibrium outcomes under both regimes, we turn now to a comparison of these outcomes. We employ three criteria: (i) the investor's expected utility in equilibrium, (ii) the fund's expected fees in equilibrium, and (iii) the standard deviation, or "volatility," of the net-of-fees returns to the investor in equilibrium.

We have already described closed-form expressions for the expected utility and expected fees in equilibirum under either regime for any fixed a. We now do the same for equilibrium volatilities. As above, let $\xi = \pi_h^2 + \pi_h\pi_l + \pi_l^2$. In a fulcrum fee regime, when $a < 1$, equilibrium volatility is given by

$$\sigma(R) = \frac{2a\xi\sqrt{2}}{6 + a(\pi_h - \pi_l)}, \tag{3.25}$$

while if $a > 1$, we have

$$\sigma(R) = \frac{2\sqrt{2}a\pi_h\xi}{7\pi_h + 2a\pi_h^2 - \pi_l - 2a\pi_h\pi_l}. \tag{3.26}$$

Equilibrium outcomes under an incentive-fee regime coincide with those under a fulcrum fee regime if $a < 1$. Consequently, volatility of returns under incentive fees when $a < 1$ are also given by (3.25). When $a > 1$, the equilibrium volatility under an incentive fee regime is

$$\sigma(R) \;=\; \frac{a\pi_l(4\xi + \pi_h\sqrt{6\xi})}{\sqrt{3}(\pi_h + 2\pi_l)}. \tag{3.27}$$

With these closed-form solutions in hand, we are in a position to prove the main result of this section:

Proposition 3.5 *If $a^{\max} \leq 1$, then equilibrium outcomes under incentive fees are identical to those under fulcrum fees. If $a^{\max} > 1$:*

1. *The investor's equilibrium expected utility is strictly higher in an incentive fee regime than in a fulcrum fee regime.*
2. *Equilibrium volatility of gross returns (i.e., of the fund portfolio) is never higher under an incentive-fee regime than under a fulcrum fee regime.*
3. *Provided a mild parameter restriction is satisfied:*
   (a) *The fund's expected fee is also strictly higher in an incentive fee regime.*
   (b) *The volatility of net-of-fees returns to the investor is strictly smaller under an incentive fee regime.*

Remark The required restriction on parameter values is stated precisely in the proof of the proposition. The restriction is a very mild one, and is effectively just a requirement that the ratio π_h/π_l not be too close to unity. Of course, if $\pi_h/\pi_l = 1$, then the benchmark portfolio would be dominated by the riskless asset, since it would then provide the same expected return at a higher volatility. □

Proof See Appendix C.5. □

Tables 1 and 2 provide a numerical illustration of Propositions 3.1–3.5. We take a specific parametrization of the benchmark returns, and present the equilibrium outcomes for various values of a ranging between $a = 0.25$ and $a = 2$. Each table presents six quantities of interest: (i) the values of b_1 and b_2 in the equilibrium fee structure, (ii) the expected fee EF received by the fund, (iii) the amount of wealth x allocated to the fund, (iv) the expected net-of-fees returns ER to the investor, (v) the volatility $\sigma(R)$ of this return, and (vi) the investor's expected utility EU.

Since the equilibria under the two regimes coincide when $a^{\max} \leq 1$, we focus in the remainder of this subsection only on the case $a^{\max} > 1$. The tables illustrate the Pareto improvement that takes place under incentive fees. For the parameterizations used, the investor's utility is a little bit higher under incentive fees than under fulcrum fees; the fund's payoff is substantially improved (by over 20%). For any fixed $a > 1$, the amount x invested in the fund is also larger under an incentive fee than under a fulcrum fee. Finally, the expected net-of-fees return to the investor is smaller under an incentive fee; but the volatility is lower, sufficiently lower, in fact, that the investor's overall welfare is improved.

To close this section, Table 3 provides further numerical illustrations of the improvement created under an incentive fee regime. A range of values is considered for the free parameters γ, π_h, and π_l. Attention is confined in the table to the case $a > 1$. As with the earlier parametrization, the fund registers a significant improvement in its welfare under a fulcrum fee regime; the gain to the investor is also positive but smaller.

3.4 Equilibrium when Fee Structure is Unrestricted

Several other fee structures than the fulcrum- and incentive-fee mechanisms may be envisaged. The most general of these is one in which the fund elects to charge a different fee in each state of the world, i.e., in which the choices of F_h, F_m, and F_l are totally unrestricted. Such unrestricted fee structures often lead to unrealistically complex state-dependence in contracting problems. Nonetheless, it is worthwhile considering this case for completeness, and also to be able to relate the earlier outcomes to it.

Unfortunately, we were not able to obtain closed-form solutions for the equilibrium in this case. In Table 4, we present, for a specific parametrization, solutions obtained using an optimzation package. As the table reveals, the unrestricted optimal fee structure involves positive fees in only a single state of the world, with zero fees in the other two states. (In particular, base or guaranteed fees are not optimal in our framework.)

More interesting is the comparison with the restricted fee structures. The unrestricted equilibrium fee structure differs from the equilibrium fee structure under both the fulcrum-fee and incentive-fee regimes when $a \leq 1$. However, when $a > 1$, outcomes under the unrestricted fee coincide with those under incentive-fees.

3.5 Some Intuition for the Results

Although the model we study differs in some important respects from traditional principal/agent models, the results of this section may be understood using intuition gained from the study of the latter. It is typically the case in principal/agent models that, under suitable regularity conditions, equilibrium payoffs to the agent are increasing in the size of the output generated. In the model studied in this paper, the same feature is reflected in the fact that in the unrestricted model, the fund only takes a fee in the state h where gross returns on its portfolio are highest; in the states m and l, we have $F_m = F_l = 0$.

However, benchmarking makes such a fee structure impossible when $a < 1$. In this case, the fund portfolio returns are always strictly less than the market portfolio returns in the state h. This means in a fulcrum fee regime we must have $F_h \leq F_m \leq F_l$ with strict inequality holding if $b_2 > 0$; while in an incentive fee regime we must have $F_h = F_m \leq F_l$ with the latter being a strict inequality if $b_2 > 0$. In either case, the fund cannot receive its highest fees in state h. Thus, outcomes are strictly worse under benchmarking when $a < 1$, regardless of whether the restricted fee structure is a fulcrum fee or an incentive fee.[8] When $a > 1$, however, the state in which the fund portfolio has its highest returns is also the state in which it exceeds the benchmark by the largest margin. Under incentive fees, this enables the fund to achieve the same result as under unrestricted fees. The improvement is also substantial under fulcrum fees, but less marked, precisely because of the nature of the "fulcrum."

4 Introducing Additional Sources of Risk

A possible criticism of the model of the previous section is that the portfolio strategies available to the fund are somewhat restricted; in particular, the return on the fund portfolio is always perfectly correlated with the return on the benchmark portfolio. In this section, we consider a generalization of the model that admits the possibility of imperfect correlation. We consider a four-state model with two risky securities and one riskless security. The riskless security will, as usual, be assumed to have a net return of zero. Letting (μ_1, μ_2) denote the net expected returns on the two risky securities and (σ_1, σ_2) their volatilities, the

[8]This result is closely related to a well-known result in principal/agent games that it is, in general, suboptimal to have the agent's payoff depend on random events beyond his control. See, e.g., Admati and Pfleiderer [1] who develop on this point to study the desirability of benchmarking.

gross return on the two risky securities in the four states are assumed to be given by:

	Security 1	**Security 2**
State 1	$1+\mu_1+\sigma_1$	$1+\mu_2+\sigma_2$
State 2	$1+\mu_1+\sigma_1$	$1+\mu_2-\sigma_2$
State 3	$1+\mu_1-\sigma_1$	$1+\mu_2+\sigma_2$
State 4	$1+\mu_1-\sigma_1$	$1+\mu_2-\sigma_2$

Finally, the probabilities q_i of the four states are taken to be

$$q_1 \;=\; q_4 \;=\; \frac{1+\rho}{4} \qquad q_2 \;=\; q_3 \;=\; \frac{1-\rho}{4}$$

where $\rho \in (-1,1)$. It is easily checked under this specification of returns and probabiltiies that the expected return and volatility of security i are, indeed, given by μ_i and σ_i; and that ρ is the correlation between the returns on the two securities.

Throughout this section, we concentrate on a symmetric version of this model where it is assumed that $\mu_1 = \mu_2 = \mu$ and $\sigma_1 = \sigma_2 = \sigma$. (The correlation ρ remains unrestricted, of course.) The benchmark portfolio is defined to be an equally weighted portfolio of the two securities. Thus, denoting by $\tilde{r}_1$ and $\tilde{r}_2$ the returns on the two securites, the benchmark returns $\tilde{r}_b$ are given by

$$\tilde{r}_b \;=\; \frac{1}{2}\left(\tilde{r}_1 + \tilde{r}_2\right).$$

Finally, it remains to specify the portfolio strategies available to the fund. Let a_1 and a_2 denote the fractions of initial asset value invested by the fund in each of the two risky securities, with the balance $(1-a_1-a_2)$ denoting the fraction invested in the riskless asset. We assume that a_1 and a_2 must be non-negative, and that there is a maximum leverage available to the fund, i.e., there is some $a^{\max} \geq 1$ such that any feasible pair (a_1, a_2) must satisfy

$$0 \leq a_1 \leq a_1 + a_2 \leq a^{\max}.$$

Note that given any choice of (a_1, a_2), the returns $\tilde{r}_p(a)$ on the fund portfolio are given by

$$\tilde{r}_p(a) \;=\; a_1\tilde{r}_1 + a_2\tilde{r}_2 + (1-a_1-a_2).$$

Fulcrum and incentive fee structures are defined exactly as in Section 3. Given any feasible choice of portfolio (a_1, a_2) for the fund, and any specific fee structure, a distribution is generated in the obvious way for the net-of-fees return R received by the investor per dollar invested in the fund. Using the expectation $E(R)$ and the variance $V(R)$ of R, (3.5) then defines the optimal amount invested in the fund as a function of the portfolio choice (a_1, a_2) and the fee structure. Taking this dependence into account, the fund chooses a portfolio mix and parameters for its fee structure that maximize its expected fees. All of this is conceptually straightforward but notationally cumbersone, so we avoid the details here.

The introduction of a second risky security makes this model significantly more complicated than the one studied in the previous section; it no longer looks amenable to analytical solution. We investigated it numerically, therefore, for a wide variety of possible parameters. In solving for the equilibrium, we adopted a two-step procedure analogous to the one used in the previous section. Namely, we first fixed a fraction $y \in [0, a^{\max}]$ of initial asset value that may be invested in the two risky securities combined (i.e., we assumed that a faction $(1 - y)$ must be invested in the riskless asset). Then, we maximized expected fees subject to the constraint that $a_1 + a_2 = y$. Using these maxima, we finally identified the optimal value of y for the fund.

The equilibria of this model have a number of properties that are analogous to, and generalize, those found in the simpler model of the last section. For example, fix any y and any (a_1, a_2) such that $a_1 + a_2 = y$, and consider the situation where the fund's portfolio is fixed at these levels. Then, if $y \leq 1$ (so the fund portfolio is not leveraged), the equilibrium fulcrum fee and the equilibrum incentive fees both turn out to be flat fees. However, if $y > 1$, then the equilibrium fee in either case involves a performance component.

Our most important finding is that although the fund in this model may choose a portfolio that is imperfectly correlated with the benchmark, it never does so: for any given value of y, and under either fee regime, the values of (a_1, a_2) that maximize the fund's payoffs subject to the constraint $a_1 + a_2 = y$ are always $a_1 = a_2 = y/2$.[9] Therefore, the multiple risky securities model of this section reduces to a one risky security model analogous to the one studied in the previous section.

Unsurprisingly, then, the welfare properties of the equilibria under the two regimes are very similar to those obtained in the previous section. Tables 5 and 6 present equilibrium values of all relevant quantities for two different parametrizations of the problem. The tables show, in particular, that

[9] It is possible to prove this result analytically in the fulcrum fee model, but an analogous proof in the incentive fee case seems very difficult.

1. When $a^{\max} \leq 1$, a flat fee with no performance component is arises as the equilibrium fee under both regimes. Thus, equilibrium outcomes coincide completely.

2. When $a^{\max} > 1$, the incentive fee regime Pareto-dominates the fulcrum fee regime, providing higher payoffs for both the fund and the investor. Volatility of net-of-fees returns are also strictly lower under incentive fees.

Thus, even under the richer set of portfolio strategies available to the fund, it continues to be the case that the investor is weakly (and, if $a^{\max} > 1$, strictly) better off under incentive fees.

5 Introducing Fund Manager Effort into the Model

This section builds on the model of Section 3 in a different direction. We investigate here the consequences of admitting moral hazard into the problem, i.e., of assuming that, apart from choice of portfolio, the fund can further affect return characteristics by expending costly "effort." The consideration of this dimension achieves two objectives. First, it ensures that our results are not driven by the ignoring of the role of effort incentives. Second, it is of independent interest to see whether better equilibria arise under this modelling extension.

Recall that in the model of section 4, when the fund picks an action $a \in [0, a^{\max}]$, the returns on its portfolio are determined as

$$\tilde{r}_p(a) = \begin{cases} 1 + \pi_h a, & \text{with probability } 1/3 \\ 1, & \text{with probability } 1/3 \\ 1 - \pi_l a, & \text{with probability } 1/3 \end{cases}$$

We now assume that returns within each state of the world are also influenced by the fund's effort level (denoted e), and are given by

$$\tilde{r}_p(a, e) = \begin{cases} 1 + \pi_h a(1+e), & \text{with probability } 1/3 \\ 1, & \text{with probability } 1/3 \\ 1 - \pi_l a(1-e), & \text{with probability } 1/3 \end{cases} \qquad (5.28)$$

We assume throughout that $e \in [0, 1]$. Observe that under this specification, it is the case that as the fund increases its effort level, the returns in states h and l both improve, while the return in state m is unaffected. Thus, more effort is unabiguously a good thing from the point of view of the resulting return distribution.[10] However, we assume that effort also has a cost $k(e)$ that must be borne by the fund. Thus, the fund's final payoffs are now given by its expected fees less the cost $k(e)$ that it incurs on effort.

Given any choice of portfolio a, effort e, and fee structure (F_h, F_m, F_l), the net-of-fees returns to the investor are determined as

$$\begin{aligned} R_h &= (1 - F_h)(1 + \pi_h a(1 + e)) \\ R_m &= (1 - F_m) \\ R_l &= (1 - F_l)(1 - \pi_l a(1 - e)) \end{aligned}$$

Using the distribution of these returns, we can identify, via (3.5), the optimal amount x^* to be invested in the fund as a function of e, a, and the fee structure. In turn, this enables a representation of fund's expected payoff (expected fees less cost of effort) in terms of these variables. Maximizing these expected payoffs then delivers the equilibrium.

We solved the model numerically when effort is assumed to be quadratic ($k(e) = ke^2$ for some $k > 0$). The results are presented in Tables 7 and 8 for fulcrum and incentive fees, respectively, with Table 9 highlighting the differences in outcomes. A perusal of these tables reveals that, even with the additional dimension of effort, the qualitative nature of the results is unchanged from Section 4. In particular:

1. When $a^{\max} \leq 1$, a flat fee arises as the equilibrium fee structure under both regimes. Thus, equilibrium outcomes coincide in theis case.

2. When $a^{\max} > 1$, a performance-dependent component appears under both regimes. The outcome under incentive fees Pareto-dominates that under fulcrum fees: both the investor and the fund are better off. Moreover, the equilibrium volatility of net-of-fees returns to the investor is lower under incentive fees.

Finally, we compare the model with effort to the model without effort. Table 9 summarizes the relevant information for three choices of the cost-of-effort parameter k: a "small" value ($k = 5$), a "medium" value ($k = 10$), and a "high" value ($k = +\infty$). The last of these corresponds to the model without effort, since it is never optimal for the fund to take positive levels of effort in this case. It is seen from the table that as the cost of effort increases, equilibrium payoffs are lowered for both the fund and the investor, reaching their lowest value when effort is infinitely costly.

[10] One can, for example, think of effort in this model as research that provides information to the fund enabling it to better allocate the fraction a of initial asset value among the different risky securities that go into constructing the benchmark portfolio.

In summary, effort appears to be a second-order effect, and our results are robust to its introduction in the model. Its main effect is in accentuating the attractiveness of incentive fee structures over fulcrum fee structures.

6 Conclusions

Existing analyses of mutual funds have mostly been conducted within a classical principal/agent framework. In this paper, we proposed an alternative model for the study of these institutions. This model was used to study the existing regulations that require fee structures used to compensate mutual fund advisers to be of the "fulcrum" variety, i.e., that decrease in the same way for underperforming an index as they increase for outperforming it.

We found little justification for the legal restrictions. In particular, we found that "incentive-fee" structures—in which the adviser receives a base fee plus a bonus for exceeding a benchmark index—Pareto-dominate fulcrum fees, providing a higher utility to all participants with, in fact, a lower level of equilibrium volatility. These results contrast with those obtained using a partial equilibrium framework in which the investor's reactions are not explicitly modelled.

Our model also provides some insight into existing fee structures in the asset-management industry. In the mutual fund industry, the most commonly used fee structure observed in practice is a flat "fraction-of-funds" fee with no explicit performance component. In hedge funds, which are not subject to the fulcrum fee requirement and which also tend to use leveraged strategies, the most common fee found in practice is an incentive fee with a large performance component. These are exactly the results that arise as equilibria in our model. In the absence of leveraging, we find that the equilibrium fulcrum fee is always a flat fee with no performance component. When incentive fees are allowed and leveraging is permitted, we find that the equilibrium fee is an incentive fee with a large performance component.

A A Brief History of the Investment Advisers Act

The Investment Advisers Act of 1940 lays out compensation strucutures that are impermissible for investment advisers. The act prohibits a registered investment adviser from receiving compensation on the basis of a share of capital gains in, or capital appreciation of, a client's account. In particular, performance-based compensation structures such as those which pay a flat fee plus a bonus for outperforming an index are disallowed.

The Act was prompted more by concerns about the inherent nature of such "incentive fees," than any evidence of actual abuse. Nonetheless, the prohibition in original act was not absolute. Incentive fees were allowed in contracts between investment advisers and investment companies (including mutual funds), as long as the chosen basis of compensation was adequately disclosed to the shareholders.

In the 1960's, this situation was challenged by the SEC, which recommended that the prohibition on incentive-fee contracts be extended to cover investment company contracts also. The commission furnished Congress with the information that of 137 registered investment companies that then had fee arrangements based in some measure on performance, 48 allowed the investment adviser to earn a bonus for good performance without a penalty for bad performance, while a further 45 had arrangements in which the rewards for superior performance far outweighed the penalties for inferior performance. Although the commission did not present Congress with any actual evidence of abuse, Congress nevertheless accepted the commission's recommendation in 1970, and amended the 1940 Act to include investment company contracts also.

At the same time, however, Congress provided for one important exemption to the prohibition of performance-based fees. Contracts with registered investment companies were allowed to have compensation based on performance if they were of the "fulcrum" variety, that is if mangerial compensation were computed symmetrically around a chosen benchmark, decreasing for underperforming the benchmark in the same way in which it increased for outperforming it.

Since 1970, there has been only one major change to the regulation of performance-based compensation. In 1985, the SEC allowed the unlimited use of performance-based fees in contracts in which the client had either (i) at least \$500,000 under the adviser's management, or (ii) a net worth of at least \$1,000,000. This amendment has not, however, affected mutual funds in any important way, since for a mutual fund to qualify for the exemption, *every* single shareholder in the fund would have to meet one of the two specified criteria.

B Existing Patterns of Fees

The single most prominent (and perhaps most intriguing) fact concerning compensation structures in the mutual fund industry is the overwhelming popularity of "fraction of funds" fees, in which the investment adviser receives as compensation a fixed fraction of the total funds under management. A recent article in the *New York Times* reported that out of 5,400 stock and bond funds, only 75 (or 1.4%) use fulcrum fees that depend non-trivially on performance.[11] Although small, the list does include some prominent names, such as Fidelity's Magellan Fund, and Vanguard's Windsor Fund.

Within each of these two categories, a number of variants may be found in the mutual fund industry. In the use of fraction-of-funds fees, for instnace, some funds tend to use a fixed percentage of assets under management, while others tend to use a sliding scale, with the percentage declining as the assets under management increases.

A typical fulcrum fee in the industry takes on the form of a base fee plus a "performance adjustment" for exceeding or falling short of a chosen benchmark. For equity funds, the benchmark is usually, though not always, the S&P 500 index. In most cases, a cap and floor are also placed on the fulcrum fee, that is, the performance adjustment component of the total fee is limited to some maximum amount (often $\pm$ 20 basis points). A variant on these themes is offered by Vanguard's Windsor fund in which the final fee is calculated as the base fee *times* an adjustment factor, where the adjustment factor varies from 0.50 to 1.50 depending on the fund's performance vis-a-vis the S&P 500 index. Finally, it is not uncommon for funds using a fulcrum fee to base the performance adjustment component not just on peformance over the last year, but over a longer period (say, the preceding three years).

C Proofs

C.1 Proof of Proposition 4.1

For ease of reference, we begin with a statement of the problem. To this end, recall that the lower bound on fees in any state is zero, which implies a corresponding upper bound of $2b_1$. Taking these bounds into account, for any portfolio choice a and any choice of $b_1 \geq 0$ and

[11] See Carole Gould, "Paying Fund Managers with Carrots and Sticks," *New York Times*, February 9, 1997. The article attributed these statistics to Lipper Analytical Services.

$b_2 \geq 0$, the fund's fees in the three states are given by

$$\begin{aligned} F_h &= \max\{0, \min\{2b_1, b_1 + b_2(a-1)\pi_h/(1+a\pi_h)\}\} \\ F_m &= b_1 \\ F_l &= \max\{0, \min\{2b_1, b_1 - b_2(a-1)\pi_l/(1-a\pi_l)\}\} \end{aligned} \tag{C.1}$$

Given the fee distribution (F_h, F_m, F_l), the net-of-fees returns R to the investor are realized as

$$\begin{aligned} R_h &= (1-F_h)(1+a\pi_h) \\ R_m &= (1-F_m) \\ R_l &= (1-F_l)(1-a\pi_l) \end{aligned} \tag{C.2}$$

The investor's optimal action now is to invest an amount x^* with the fund, where

$$x^* = \frac{E(R)-1}{\gamma V(R)}. \tag{C.3}$$

Note that x^* is a function of only γ and the three parameters (a, b_1, b_2) chosen by the fund. The expected fees received by the fund are

$$EF = \frac{1}{3}[F_h(1+a\pi_h) + F_m + F_l(1-a\pi_l)] \cdot x^*. \tag{C.4}$$

For any fixed a, the fund's objective is to choose (b_1, b_2) so as to maximize these expected fees. The tricky part of this maximization exercise is ensuring that all the constraints are met, i.e., that (i) b_1 and b_2 are non-negative, and (ii) equilibrium fees in any state do not fall below the floor level or exceed the ceiling level.

We begin by checking for the existence of an "unconstrained" solution. That is, we ignore the non-negativity constraints on b_1 and b_2, as well as the floor and ceiling levels, and simply maximize the expected fees function with respect to (b_1, b_2). Taking the first-order condition with respect to b_1 results in two possible values for b_1 in terms of b_2:

$$b_1 = 1 - b_2\left(\frac{a-1}{a}\right) \tag{C.5}$$

$$b_1 = \frac{(a-b_2(a-1))(3a-6b_2(a-1)+a^2(\pi_h-\pi_l)(\pi_h-\pi_l)}{(6a-6b_2(a-1)+a^2(\pi_h-\pi_l))(3+a(\pi_h-\pi_l))} \tag{C.6}$$

Under the first case, we have $b_2 = a(1-b_1)/(a-1)$. Substituting this into the expression for the fees in each state yields

$$\begin{aligned} F_h &= (b_1 + a\pi_h)/(1+a\pi_h) \\ F_m &= b_1 \\ F_l &= (b_1 - a\pi_l)/(1-a\pi_l) \end{aligned}$$

which, in turn implies, that we must have $R_h = R_m = R_l = (1-b_1)$. As long as $b_1 > 0$, this ensures that the investment with the fund will never be a positive amount, ruling this out as a candidate solution.

The second candidate (C.6) may also be ruled out as a solution. If we use this expression to substitute for b_1 in terms of b_2 in the fees, and then maximize expected fees (ignoring the constraints), the only value of b_2 that meets the first-order conditions implies an infinite value for b_1. Thus, no unconstrained solutions exist.

We turn to constrained solutions. There are four constraints that could hold with equality in a solution: (i) $b_1 = 0$, (ii) $b_2 = 0$, (iii) the floor of zero could hold for fees in some state, and (iv) the ceiling of $2b_1$ could hold for fees in some states. It helps in the sequel to consider the cases $a < 1$ and $a > 1$ separately.

C.1.1 The Case $a < 1$

When $a < 1$, it is the case that in state h the fund portfolio underperforms the benchmark, while in state l the fund portfolio outperforms the benchmark. This means for any non-negative values of b_1 and b_2, the fees are effectively given by

$$\begin{aligned} F_h &= \max\{0, b_1 + b_2(a-1)\pi_h/(1+a\pi_h)\} \\ F_m &= 1 - b_1 \\ F_l &= \min\{2b_1, b_1 - b_2(a-1)\pi_l/(1-a\pi_l)\} \end{aligned}$$

To identify the optimal choice of (b_1, b_2), we proceed in several steps, identifying at each step the set of candidate solutions that arise when only a subset of the constraints holds with equality. Comparing the expected fees in the candidate solutions then yields the optimal choice of (b_1, b_2). We first examine the candidate solutions that arise when either $b_1 = 0$ or $b_2 = 0$. Then, we will identify the candidate solutions that arise if the lower-bound constraint on F_h holds with equality. Third, we will repeat this exercise when the upper-bound constraint on F_l holds with equality.

So consider first the case $b_1 > 0$, $b_2 = 0$. Then, we have $F_h = F_m = F_l = 1 - b_1$. We use this to obtain first the investor's optimal response as a function of b_1, and thereby the fund's expected fees as a function of b_1. Maximizing the expected fees over b_1 results in only a single possibility, namely

$$b_1 = \frac{a(\pi_h - \pi_l}{6 + a(\pi_h - \pi_l)} \tag{C.7}$$

It is easily checked that all the relevant constraints are met when b_1 is given by (C.7) and $b_2 = 0$. Thus, the expected fees under the choice (C.7) are a candidate solution to th efund's optimization problem. Letting $\xi = \pi_h^2 + \pi_h \pi_l + \pi_l^2$, these fees are given by

$$EF = \frac{(3 + a(\pi_h - \pi_l))(\pi_h - \pi_l)^2}{24\gamma\xi}. \tag{C.8}$$

The case $b_1 = 0$, $b_2 > 0$ is easily eliminated from consideration: in this case, fees must violate the non-negativity condition in state h and the upper bound of $2b_1$ in state l.

We turn now to the case where the lower bound on F_h holds with equality. If $F_h = 0$, then

$$b_1 = -b_2(a-1)\left(\frac{\pi_h}{1 + a\pi_h}\right).$$

We can use this expression to obtain the entire fee structure in terms of b_2. Thus, we can identify the investor's optimal action as a function solely of b_2, and thereby the fund's expected fees. Taking the first-order condition with respect to b_2 of these expected fees results in only one non-negative value for b_2. This value of b_2, and the corresponding value of b_1 are:

$$b_1 = \frac{a\pi_h(\pi_h - \pi_l}{5\pi_h + \pi_l} \tag{C.9}$$

$$b_2 = \frac{-a(1 + a\pi_h)(\pi_h - \pi_l)}{(5\pi_h + \pi_l)(a-1)}. \tag{C.10}$$

It is easy to check that under these values of b_1 and b_2, the non-negativity requirements as well as the upper-bound condition for F_l are satisfied. Thus, (C.9)–(C.10) are also candidate solutions to the optimization problem. Letting $\xi = (\pi_h^2 + \pi_h \pi_l + \pi_l^2)$ as above, the expected fees they generate is given by

$$EF = \frac{(\pi_h - \pi_l)^2(2\pi_h + \pi_l)}{24\pi_h\gamma\xi}. \tag{C.11}$$

Next, we turn to the case where the upper-bound constraint in F_l holds with equality. If $F_l = 2b_1$, then we must have

$$b_1 \;=\; -b_2(a-1)\left(\frac{\pi_l}{1-a\pi_l}\right).$$

Once again, we can use this expression to obtain the entire fee structure in terms of b_2. Thus, we can identify the investor's optimal action as a function solely of b_2, and thereby the fund's expected fees. Taking the first-order condition with respect to b_2 of these expected fees results in only one non-negative value for b_2. This value, and the corresponding value it implies for b_1, are

$$b_1 \;=\; \frac{a\pi_l(\pi_h-\pi_l)}{7\pi_l-2a\pi_l^2+2a\pi_h\pi_l-\pi_h} \tag{C.12}$$

$$b_2 \;=\; \frac{a(1-a\pi_l)(\pi_h-\pi_l)}{(1-a)(7\pi_l-2a\pi_l^2+2a\pi_h\pi_l-\pi_h)} \tag{C.13}$$

Expressions (C.12)–(C.13) offer a third pair of candidate solutions. Defining ξ as above, the expected fees they imply is given by

$$EF \;=\; \frac{(\pi_h-\pi_l)^2(4\pi_l-2a\pi_l^2+2a\pi_h\pi_l-\pi_h)}{24\gamma\pi_l\xi}. \tag{C.14}$$

As the last step in the proof, we compare the expected fees under the three candidate solutions. Consider first (C.8) and (C.11). Eliminating the common terms, it is seen that the former expression is larger than the latter only if

$$3\pi_h + a\pi_h(\pi_h-\pi_l) \;>\; 2\pi_h+\pi_l,$$

which always holds since $\pi_h > \pi_l > 0$. Now comparing (C.8) with the expected fee (C.14) under the third candidate solution, the former is seen to be larger if and only if

$$(1-a\pi_l)(\pi_h-\pi_l) \;\geq\; 0,$$

which always holds under our assumptions on π_h and π_l. Thus, the largest of the three candidate values for the expected fee is given by (C.8), and it follows that when $a < 1$, the unique optimal fee structure for the fund is to set $b_2 = 0$ and have b_1 given by (C.7).

C.1.2 The case $a > 1$

When $a > 1$, the fund portfolio always does worse than the benchmark portfolio in the state l and always does better in the state h. Therefore, the lower bound of zero can have an impact only in state l and the higher bound of $2b_1$ can have an impact only in state h; the effective constrained fee structure is given by

$$\begin{aligned} F_h &= \min\{2b_1, b_1 + b_2(a-1)\pi_h/(1+a\pi_h)\} \\ F_m &= b_1 \\ F_l &= \max\{0, b_1 + b_2(a-1)\pi_l/(1-a\pi_l)\}. \end{aligned}$$

We proceed in the same manner that we did above by identifying candidate solutions when some constraints hold with equality. The first case we consider, that of $b_1 > 0$ and $b_2 = 0$, again results in the candidate solution (C.7) with an expected fee given by (C.8). The second case, that of $b_1 = 0$ and $b_2 > 0$, can be eliminated from consideration for the same reason as above.

Consider next the case where the lower bound on F_l holds with equality. If $F_l = 0$, then we must have

$$b_1 = b_2(a-1)\left(\frac{\pi_l}{1-a\pi_l}\right).$$

We can use this to eliminate b_1 from the expression for expected fees in the usual manner. Taking the first-order conditions of the expected fees with respect to b_2 now results in two solutions for b_2. The first solution, together with its corresponding value of b_1, is:

$$b_1 = a\pi_l \tag{C.15}$$

$$b_2 = \frac{a(1-a\pi_l)}{a-1} \tag{C.16}$$

Under these values for (b_1, b_2), a simple calculation reveals that the net-of-fees returns to the investor satisfy $R_h = R_m = R_l = (1 - a\pi_l)$. It is apparent that optimal investment in the fund cannot be positive under these circumstnaces, ruling this out as a candidate solution.

The second solution when $F_l = 0$ is given by

$$b_1 = \frac{a\pi_l(\pi_h - \pi_l)}{\pi_h + 5\pi_l} \tag{C.17}$$

$$b_2 \;=\; \frac{a(\pi_h-\pi_l)(1-a\pi_l)}{(a-1)(\pi_h+5\pi_l)}. \tag{C.18}$$

This pair of values also satisfies the upper-bound on F_h if $(\pi_h-\pi_l)\le 2a\pi_h\pi_l$. We retain it, therefore, as a candidate solution subject to this condition being met. Defining ξ as above, the expected fee in this case is

$$EF \;=\; \frac{(\pi_h-\pi_l)^2(\pi_h+2\pi_l)}{24\gamma\pi_l\xi} \tag{C.19}$$

Next, we turn to the case where the upper bound on F_h holds with equality. If $F_h = 2b_1$, then we must have

$$b_1 \;=\; b_2(a-1)\left(\frac{\pi_h}{1+a\pi_h}\right).$$

We use this in the usual manner to eliminate b_1 and obtain an expression for the expected fees in terms of b_2. Taking the first-order conditions of these expected fees with respect to b_2 gives us two possible values for b_2. One of these, together with its corresponding value for b_1 results in the net-of-fees returns to the investor of $R_h = R_m = R_l = (1+a\pi_h)/(1+2a\pi_h) < 1$. This eliminates it as a candidate solution. The other value for b_2, together with its corresponding value for b_1, is

$$b_1 \;=\; \frac{a\pi_h(\pi_h-\pi_l)}{7\pi_h+2a\pi_h 2-\pi_l-2a\pi_h\pi_l} \tag{C.20}$$

$$b_2 \;=\; \frac{a(\pi_h-\pi_l)(1+a\pi_h)}{(a-1)(7\pi_h+2a\pi_h^2-\pi_l-2a\pi_h\pi_l)}. \tag{C.21}$$

This is a third pair of candidate solutions with an expected fee of

$$EF \;=\; \frac{(\pi_h-\pi_l)^2(2a\pi_h^2-\pi_l+4\pi_h-2a\pi_h\pi_l)}{24\gamma\pi_h\xi}. \tag{C.22}$$

To complete the proof, we compare the expected fees (C.8), (C.19), and (C.22). The last of these exceeds the first if and only if $(\pi_h-\pi_l)(1+a\pi_h) > 0$ which always holds. It also exceeds the second if $2a\pi_h\pi_l-(\pi_h-\pi_l) > 0$. But the second fee is a candidate solution only if this condition holds (see above). Therefore, the third candidate solution dominates the others, and the equilibrium when $a > 1$ is given by (C.20)–(C.21) with an expected fee of (C.22).

C.1.3 The Case $a = 1$

We have not so far considered the case $a = 1$, but this is easily accomodated within the developments so far. When $a = 1$, the fund portfolio and the benchmark portfolio have identical returns. Thus, b_2 is irrelevant, and it may be set to zero without loss. Maximizing expected fees with respect to b_1 results in the solution (C.7) for b_1 with an expected fee of (C.8). This completes the proof of Proposition 4.1. □

C.2 Proof of Proposition 4.2

When $a \leq 1$, we have seen that the expected fee is given by (C.8), which is clearly increasing in a. Thus, $a^* = a^{\max}$ is the optimal action if $a^{\max} \leq 1$. If $a > 1$, the expected fee is given by (C.22) which is also increasing in a. Moreover, we have shown that for any fixed $a > 1$, (C.22) is larger than (C.8). Since (C.8) is increasing in a, it is the case that for any $a > 1$, the maximum expected fee (C.22) under a dominates the expected fee (C.8) for any $a \leq 1$. It is immediate now that the optimal action for any $a^{\max} > 1$ is $a^* = a^{\max}$, completing the proof. □

C.3 Proof of Proposition 4.3

The maximization problem here is the same as was outlined at the top of section C.1, with the difference being that the fee structure in effect is an incentive fee structure. Thus, the non-negativity constraints on the realized fees, or the corresponding upper-bounds, play no role here. The only constraints facing the fund in its choice of parameters for the fees are that b_1 and b_2 must be non-negative.

The proof provided in section C.1 concerning the absence of unconstrained solutions evidently continues to apply to this case also. Thus, only two forms of constrained solutions are possible: $b_1 > 0$, $b_2 = 0$ and $b_1 = 0$, $b_2 > 0$. It helps, in identifying the optimal solution, to handle the cases $a < 1$ and $a > 1$ separately.

C.3.1 The Case $a < 1$

For any $a < 1$, the fund portfolio outperforms the benchamrk in state l and underperforms it in state h. Therefore, given any choices of b_1 and b_2, the fund's fees in the three states are determined as

$$\begin{aligned} F_h &= b_1 \\ F_m &= b_1 \\ F_l &= b_1 + b_2(a-1)\pi_l/(1-a\pi_l) \end{aligned}$$

Consider first the case where $b_1 > 0$ and $b_2 = 0$. In this case, solving for the optimal value of b_1 evidently results in the same solution as in the corresponding fulcrum fee case: the optimal b_1 is given by (C.7) and the expected fee is given by (C.8).

Now consider $b_1 = 0$. Solving for the expected fee in terms of b_2 using the procedure we have described many times above, and then taking the first-order conditions with respect to b_2 results in only a single non-negative solution for b_2. Defining ξ once again as $(\pi_h^2 + \pi_h\pi_l + \pi_l^2)$, this solution is given by:

$$b_2 = \frac{a(\xi - \pi_h\sqrt{3\xi}}{(a-1)\pi_l(2\pi_h + \pi_l)} \tag{C.23}$$

It is easily checked that b_2 so defined is strictly positive and is therefore a candidate solution. The expected fee it generates is

$$EF = \frac{(\sqrt{3\xi} - 3\pi_h)(\xi - \pi_h\sqrt{3\xi})}{6\gamma\pi_h(2\xi + \pi_l\sqrt{3\xi})} \tag{C.24}$$

To complete the proof for the case $a < 1$, we compare expected fees under the two candidate solutions (C.8) and (C.24). Let π_h/π_l be denoted by k, and define $d = 1 + k + k^2$. Note that $k > 1$. Dividing (C.8) by (C.24), we now obtain:

$$\text{ratio} = \frac{k(k-1)^2(3 + a\pi_l(k-1))(2d + \sqrt{3d})}{4d(\sqrt{3d} - 3k)(d - k\sqrt{3d})}.$$

Both numerator and denominator are positive; since $k > 1$, this ratio is an increasing function of a, and reaches its minimum when $a = 0$. When $a = 0$, a plot of the ratio reveals that it is strictly increasing in k and greater than unity whenever $k \geq 1$. Thus, the ratio is also greater than unity for $a > 0$ when $k > 1$. It follows that the optimal incentive fee structure when $a < 1$ is to have $b_1 > 0$ and $b_2 = 0$, with b_1 given by (C.7).

C.3.2 The Case $a > 1$

When $a > 1$, the fund portfolio outperforms the benchmark in state h and underperforms it in state l. Therefore, for any choice of (b_1, b_2), the fund's fees are given by

$$\begin{aligned} F_h &= b_1 + b_2(a-1)\pi_h/(1+a\pi_h) \\ F_m &= b_1 \\ F_l &= b_1 \end{aligned}$$

Once again, there are two cases to consider: when $b_1 = 0$ and when $b_2 = 0$. In the former case, we clearly get the solution (C.7) for b_1 with expected fees given by (C.8). In the latter case, obtaining the expected fees in terms of b_2 and taking first-order conditions results in two possible values for b_2:

$$b_2 = \frac{a(\xi - \pi_l\sqrt{3\xi})}{(a-1)\pi_h(\pi_h + 2\pi_l)} \tag{C.25}$$

$$b_2 = \frac{a(\xi + \pi_l\sqrt{3\xi})}{(a-1)\pi_h(\pi_h + 2\pi_l)} \tag{C.26}$$

The expected fees in the two solutions are, respectively:

$$F = \frac{(\sqrt{3\xi} - 3\pi_l)(\xi - \pi_l\sqrt{3\xi})}{6\gamma\pi_l(2\xi + \pi_h\sqrt{3\xi})} \tag{C.27}$$

$$F = \frac{(-\sqrt{3\xi} - 3\pi_l)(\xi + \pi_l\sqrt{3\xi})}{6\gamma\pi_l(2\xi - \pi_h\sqrt{3\xi})} \tag{C.28}$$

The first of these expressions is positive, but the second can be shown to be negative, so we discard it as a possible solution.

To complete the proof, it remains to compare the expected fees (C.27) to the expected fees (C.8) under the two solutions. Once again, we define $k = \pi_h/\pi_l$ and $d = 1 + k + k^2$, and take the ratio of (C.28) to (C.8). Cancelling common terms, this ratio is seen to be

$$\text{ratio} = \frac{4d(d - \sqrt{3d})(\sqrt{3d} - 3)}{(k-1)^2(3 + k(a-1)\pi_l)(2d + k\sqrt{3d})}$$

The ratio is decreasing in a and reaches its minimum value at the maximum feasible value for a of $1/\pi_l$. A plot of the ratio shows that even when $a = 1/\pi_l$, the ratio is strictly greater than unity whenever $k > 1$. It follows that the optimal structure of incentive fees when $a > 1$ has $b_1 = 0$ and $b_2 > 0$, with the optimal b_2 given by (C.25), and the expected fees under this optimal choice given by (C.27).

C.3.3 The Case $a = 1$

As with fulcrum fees, the case $a = 1$ is easily handled.The fund portfolio and the benchmark portfolio have identical returns in this case. Thus, b_2 is irrelevant, and it may be set to zero without loss. Maximizing expected fees with respect to b_1 results in the solution (C.7) for b_1 with an expected fee of (C.8). This completes the proof of Proposition 4.3. □

C.4 Proof of Proposition 4.4

When $a \leq 1$, expected fees in equilibrium are given by (C.8) which is clearly increasing in a. Thus, if $a^{\max} \leq 1$, the optimal value of a for the fund is $a^{\max}$. If $a > 1$, we have shown that equilibrium fees are given by (C.8), which is independent of a. In the course of establishing this result, we also showed that for any fixed $a > 1$ the expected fees (C.27) are greater than (C.8); the latter, in turn, is greater than (C.8) for any $a \leq 1$. It follows that when $a^{\max} > 1$, any value of $a > 1$ is an optimal action for the fund, completing the proof of Proposition 4.4.
□

C.5 Proof of Proposition 4.5

When $a^{\max} \leq 1$, the equilibrium strategies under fulcrum and incentive fees coincide, so the equilibrium payoffs to the investor and fund are identical under the two regimes, as are equilibrium risk levels. To complete the proof we will consider the case $a^{\max} > 1$ and show that

1. The equilibrium expected utility under incentive fees is always greater than under fulcrum fees.
2. The volatility of the fund portfolio is never lower under fulcrum fees than under incentive fees.
3. Provided π_h/π_l is sufficiently large (precise bounds are given below):
   (a) The expected fee under an incentive fee regime dominates that under a fulcrum regime.
   (b) The volatility of net-of-fees returns is strictly lower under incentive fees than under fulcrum fees.

We begin with a comparison of the equilibrium expected utilities under the two regimes. Fix any $a > 1$. Recall that for $a > 1$, the equilibrium expected utility is independent of a

in both cases. Using the closed-forms described in the text, the ratio of the expected utility under an incentive fee to that under a fulcrum fee is given by

$$\text{Ratio of EU} = \frac{4\xi[(1+8\gamma)\xi + 3\pi_l^2 - 2\sqrt{3\xi}(\pi_l - 2\pi_h)]}{(1+16\gamma)\xi - \pi_h\pi_l)(2\xi + \pi_h\sqrt{3\xi})} \tag{C.29}$$

Let $k = \pi_h/\pi_l > 1$, and let $d = 1 + k + k^2$. Substituting this into (C.29) and eliminating common terms, we obtain:

$$\text{Ratio of EU} = \frac{4d\left[(4+k+k^2+8\gamma d) + 2\,(2\gamma k - 1)\,\sqrt{3d}\right]}{\left[(k-1)^2 + 16\gamma d\right]\left[2d + k\sqrt{3d}\right]} \tag{C.30}$$

The right hand-side is a function of just the single variable k. Plotting this right-hand side reveals that the ratio is an increasing function of k and is greater than unity for all values of $k > 1$. This completes the proof that the investor's welfare is higher under an incentive fee.

Next, note that when $a^{\max} > 1$, the fund's uniquely optimal choice of a under a fulcrum fee is $a = a^{\max}$, but any $a > 1$ is optimal for the fund under an incentive fee. Since the volatility of the fund portfolio is given by a^2, it follows immediately that this volatility is never higher (and could be strictly lower) under an incentive fee than under a fulcrum fee.

Turning to the fund's payoffs, fix any $a > 1$. Recall that the fund's expected fee for $a > 1$ is independent of a in an incentive fee regime, but not in a fulcrum fee regime. The ratio of the expected fee under incentive fees to that under fulcrum fees is given by

$$\text{Ratio of EF} = -\frac{4\pi_h\xi(-3\pi_l + \sqrt{3\xi})(\xi - \pi_l\sqrt{3\xi})}{(\pi_h - \pi_l)^2\pi_l(\pi_l + 2a\pi_h\pi_l - 4\pi_h - 2a\pi_h^2)(2\xi + \pi_h\sqrt{3\xi})} \tag{C.31}$$

Using, once again, the substitutions $k = \pi_h/\pi_l$ and $d = 1 + k + k^2$, and eliminating common terms, we obtain

$$\text{Ratio of EF} = \frac{4kd(d - \sqrt{3d})(\sqrt{3d} - 3)}{(k-1)^2(4k - 1 - 2ak\pi_l(1-k))(2d + k\sqrt{3d})}. \tag{C.32}$$

The right-hand side is greater than unity whenever

$$a\pi_l > \left(\frac{4kd(d - \sqrt{3d})(\sqrt{3d} - 3) - (k-1)^2(4k-1)(k\sqrt{3d} + 2d)}{2(k-1)^3k(2d + k\sqrt{3d})}\right) \tag{C.33}$$

This restriction holds in "most" reasonable cases, failing only when k is too close to unity and $a\pi_l$ is large (close to its upper bound). Of course, whenever it holds, the fund's equilibrium payoffs are strictly higher under incentive fees than udner fulcrum fees.

Finally, consider the volatility of the net-of-fees returns to the investor. Fix any $a > 1$. The ratio of this volatility under a fulcrum fee to that under an incentive fee is

$$\text{Ratio of Vol} = \frac{2a\pi_h(\pi_h + 2\pi_l)\sqrt{3\xi}}{(2a\pi_h^2 - \pi_l + \pi_h(7 - 2a\pi_l))(a\pi_l\sqrt{2\xi + \pi_h\sqrt{3\xi}})} \tag{C.34}$$

Expressing this ratio in terms of k and d defined as above, we have

$$\text{Ratio of Vol} = \frac{2k(2+k)\sqrt{3d}}{(6k + (k-1)(1 + 2ak\pi_l))\sqrt{2d + k\sqrt{3d}}} \tag{C.35}$$

This ratio is greater than unity whenever

$$a\pi_l > \frac{(1 - 7k)(\sqrt{2d + k\sqrt{3d}}) + 2k\sqrt{3d}(2+k)}{2k(k-1)(\sqrt{2d + k\sqrt{3d}})}. \tag{C.36}$$

As with condition (C.33), this condition also holds in "most" reasonable cases, failing only when k is too close to unity and $a\pi_l$ is large. This completes the proof of Proposition 3.5. □

D A Risk-Averse Fund

Throughout this paper, we have assumed that the fund is risk-neutral. This assumption was made in part for analytical tractability, since it enabled us to obtain closed-form solutions in Section 3. However, it turns out that, from a qualititative standpoint, the assumption is not very important: the same features of the equilibria highlighted in Section 3 also obtained when the fund's utility function was taken to be $u(x) = x^\gamma$ for $\gamma \in (0, 1]$. We present some examples of the new equilibria below for this version of the model of Section 3. The figures presented here were obtained by solving the problem numerically.

So fix any value of $\gamma \in (0, 1]$. As in Section 3, we found that whenever $a^{\max} \leq 1$, the equilibrium fee under both a fulcrum fee regime and an incentive fee regime is a flat fee. Thus, equilibrium outcomes coincide completely in this case. Table 10 presents numerical values of equilibrium outcomes under the two regimes for two values of γ and three values of $a^{\max} > 1$. As the table shows, the qualitative nature of these figures is the same as that which obtained in Section 3, with the outcomes under an incentive fee regime dominating those under a fulcrum fee regime on all counts. The only additional feature of interest is that as the value of γ falls, the difference in outcomes between the regimes narrows.

Table 1: Optimal Contracts in the Fulcrum Fee case

This table presents the equilibrium values in a fulcrum-fee regime of six quantities for various given values of the fund's portfolio choice a: (i) the fee structure b_1 and b_2, (ii) the amount x invested in the fund, (iii) the net-of-fees expected returns ER to the investor, (iv) the volatility of these returns $\sigma(R)$, (v) the fund's expected fees EF, and (vi) the investor's expected utility EU. The investor's variance-aversion parameter is fixed at $\gamma = 2$, and the parameters of the benchmark portfolio returns are fixed at $\pi_h = 0.15$ and $\pi_l = 0.05$.

a	0.25	0.5	0.75	1.00	1.25	1.5	2
b_1	0.0041	0.0083	0.0123	0.0164	0.0181	0.0215	0.0283
b_2	0.0000	0.0000	0.0000	0.0000	0.5723	0.3517	0.2453
x	4.6346	2.3269	1.5577	1.1731	1.0641	0.8932	0.6795
ER	1.0041	1.0083	1.0123	1.0164	1.0181	1.0215	1.0283
$\sigma(R)$	0.0212	0.0421	0.0630	0.0836	0.0922	0.1098	0.1443
EF	0.0194	0.0196	0.0197	0.0199	0.0251	0.0254	0.0261
EU	1.0096	1.0096	1.0096	1.0096	1.0096	1.0096	1.0096

Table 2: Optimal Contracts in the Performance Fee case

This table presents the equilibrium values of in an incentive-fee regime of six quantities for various given values of the fund's portfolio choice a: (i) the fee structure b_1 and b_2, (ii) the amount x invested in the fund, (iii) the net-of-fees expected returns ER to the investor, (iv) the volatility of these returns $\sigma(R)$, (v) the fund's expected fees EF, and (vi) the investor's expected utility EU. The investor's variance-aversion parameter is fixed at $\gamma = 2$, and the parameters of the benchmark portfolio returns are fixed at $\pi_h = 0.15$ and $\pi_l = 0.05$.

a	0.25	0.5	0.75	1.00	1.25	1.5	2
b_1	0.0041	0.0083	0.0123	0.0164	0.0000	0.0000	0.0000
b_2	0.0000	0.0000	0.0000	0.0000	2.2517	1.3510	0.9007
x	4.6346	2.3269	1.5577	1.1731	1.4508	1.2090	0.9067
ER	1.0041	1.0083	1.0123	1.0164	1.0135	1.0162	1.0216
$\sigma(R)$	0.0212	0.0421	0.0630	0.0836	0.0683	0.0819	0.1092
EF	0.0194	0.0196	0.0197	0.0199	0.0408	0.0408	0.0408
EU	1.0096	1.0096	1.0096	1.0096	1.0098	1.0098	1.0098

Table 3: Comparison of Fulcrum and Performance Fee Structures

This table examines the welfare levels of the investor and fund manager under a fulcrum fee regime and an incentive fee regime. The parameters that are varied are the investor's variance-aversion γ, and the parameters π_h and π_l of the benchmark portfolio returns. Results are provided for a range of portfolio values $a > 1$.

	Fulcrum-Fee			Incentive-Fee		
$\gamma = 2,\ \pi_h = 0.2,\ \pi_h = 0.05$						
a	EF	EU	$\sigma(R)$	EF	EU	$\sigma(R)$
1.25	0.0368	1.0134	0.1137	0.0729	1.0138	0.0730
1.50	0.0375	1.0134	0.1350	0.0729	1.0138	0.0876
1.75	0.0382	1.0134	0.1559	0.0729	1.0138	0.1023
2.00	0.0388	1.0134	0.1764	0.0729	1.0138	0.1169
$\gamma = 2, \pi_1 = 0.2, \pi_2 = 0.10$						
a	EF	EU	$\sigma(R)$	EF	EU	$\sigma(R)$
1.25	0.0112	1.0045	0.1386	0.0138	1.0045	0.1228
1.50	0.0113	1.0045	0.1651	0.0138	1.0045	0.1474
1.75	0.0115	1.0045	0.1912	0.0138	1.0045	0.1719
2.00	0.0116	1.0045	0.2169	0.0138	1.0045	0.1965
$\gamma = 10, \pi_1 = 0.15, \pi_2 = 0.05$						
a	EF	EU	$\sigma(R)$	EF	EU	$\sigma(R)$
1.25	0.0050	1.0019	0.0797	0.0082	1.0020	0.0683
1.50	0.0051	1.0019	0.1098	0.0082	1.0020	0.0819
1.75	0.0052	1.0019	0.1272	0.0082	1.0020	0.0956
2.00	0.0052	1.0019	0.1443	0.0082	1.0020	0.1092

Table 4: Optimal Contracts in the Unrestricted Fee case

This table presents the optimal values of the percentage fees F_h, F_m, and F_l for various choices of a by the fund. The investor's variance-aversion parameter is fixed at $\gamma = 2$, and the benchmark return parameters are fixed at $\pi_h = 0.15$, $\pi_l = 0.05$. The table reports the equilibrium values of: (i) the fees in each state of the world, (ii) the amount x invested in the fund, (iii) the net-of-fees expected returns ER to the investor, (iv) the volatility of these returns $\sigma(R)$, (v) the fund's expected fees EF, and (vi) the investor's expected utility EU.

a	0.25	0.5	0.75	1.00	1.25	1.5	2
F_h	0.0163	0.0314	0.0455	0.0587	0.0711	0.0827	0.1039
F_m	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F_l	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
x	7.2538	3.6269	2.4179	1.8135	1.4508	1.2090	0.9067
ER	1.0027	1.0054	1.0081	1.0108	1.0135	1.0162	1.0216
$\sigma(R)$	0.0137	0.0273	0.0410	0.0546	0.0683	0.0819	0.1092
EF	0.0408	0.0408	0.0408	0.0408	0.0408	0.0408	0.0408
$E(U)$	1.0098	1.0098	1.0098	1.0098	1.0098	1.0098	1.0098

Table 5: Two sources of risk: optimal contracts in the Fulcrum fee model

This table presents the equilibrium levels of six quantities for the three security model under a fulcrum fee regime: (i) the parameters b_1 and b_2 of the equilibrium fulcrum fee, (ii) the amount x invested in the fund, (iii) the net-of-fees expected returns ER to the investor, (iv) the volatility of these returns $\sigma(R)$, (v) the fund's expected fees EF, and (vi) the investor's expected utility EU. The fraction $(1-y)$ of initial asset value invested in the riskless asset is taken as fixed in this exercise; the table includes the optimal fractions a_1 and a_2 of initial asset value invested in each of the two risky assets. Seven values are used for y ranging from 0.25 to 2.00. The investor's variance aversion parameter is fixed at $\gamma = 2$, and the return parameters on the risky securities are fixed at $\mu_1 = \mu_2 = 0.1$ and $\sigma_1 = \sigma_2 = 0.2$. Three values are tried for the correlation in returns: $\rho = 0.3, 0, -0.3$.

Panel A: $\rho = 0.3$

$y = a_1 + a_2$	0.25	0.50	0.75	1.00	1.25	1.50	2.00
a_1	0.1250	0.2500	0.3750	0.5000	0.6250	0.7500	1.0000
a_2	0.1250	0.2500	0.3750	0.5000	0.6250	0.7500	1.0000
b_1	0.0123	0.0244	0.0361	0.0476	0.0484	0.0569	0.0732
b_2	0.0000	0.0000	0.0000	0.0000	0.8871	0.5506	0.3902
x	3.8942	1.9712	1.3301	1.0096	0.9936	0.8440	0.6571
ER	1.0123	1.0244	1.0361	1.0476	1.0484	1.0569	1.0732
$\sigma(r)$	0.0398	0.0787	0.1166	0.1536	0.1560	0.1837	0.2360
EF	0.0493	0.0505	0.0517	0.0529	0.0761	0.0785	0.0833
EU	1.0240	1.0240	1.0240	1.0240	1.0240	1.0240	1.0240

Panel B: $\rho = -0.3$

$y = a_1 + a_2$	0.25	0.50	0.75	1.00	1.25	1.50	2.00
a_1	0.1250	0.2500	0.3750	0.5000	0.6250	0.7500	1.0000
a_2	0.1250	0.2500	0.3750	0.5000	0.6250	0.7500	1.0000
b_1	0.0123	0.0244	0.0361	0.0476	0.0484	0.0569	0.0732
b_2	0.0000	0.0000	0.0000	0.0000	0.8871	0.5506	0.3902
x	7.2321	3.6607	2.4702	1.8750	1.8452	1.5675	1.2202
ER	1.0123	1.0244	1.0361	1.0476	1.0484	1.0569	1.0732
$\sigma(r)$	0.0292	0.0577	0.0855	0.1127	0.1145	0.1348	0.1732
EF	0.0915	0.0937	0.0960	0.0982	0.1414	0.1458	0.1548
EU	1.0446	1.0446	1.0446	1.0446	1.0446	1.0446	1.0446

Table 6: Two sources of risk: optimal contracts in the Performance fee model

This table presents the equilibrium levels of six quantities for the three security model under an incentive fee regime: (i) the parameters b_1 and b_2 of the equilibrium incentive fee, (ii) the amount x invested in the fund, (iii) the net-of-fees expected returns ER to the investor, (iv) the volatility of these returns $\sigma(R)$, (v) the fund's expected fees EF, and (vi) the investor's expected utility EU. The fraction $(1-y)$ of initial asset value invested in the riskless asset is taken as fixed in this exercise; the table includes the optimal fractions a_1 and a_2 of initial asset value invested in each of the two risky assets. Seven values are used for y ranging from 0.25 to 2.00. The investor's variance aversion parameter is fixed at $\gamma = 2$, and the return parameters on the risky securities are fixed at $\mu_1 = \mu_2 = 0.1$ and $\sigma_1 = \sigma_2 = 0.2$. Three values are tried for the correlation in returns: $\rho = 0.3, 0, -0.3$.

Panel A: $\rho = 0.3$

$y = a_1 + a_2$	0.25	0.50	0.75	1.00	1.25	1.50	2.00
a_1	0.1250	0.2500	0.3750	0.5000	0.6250	0.7500	1.0000
a_2	0.1250	0.2500	0.3750	0.5000	0.6250	0.7500	1.0000
b_1	0.0123	0.0244	0.0361	0.0476	0.0000	0.0000	0.0000
b_2	0.0000	0.0000	0.0000	0.0000	2.6059	1.5635	1.0424
x	3.8942	1.9712	1.3301	1.0096	1.2629	1.0524	0.7893
ER	1.0123	1.0244	1.0361	1.0476	1.0387	1.0464	1.0619
$\sigma(r)$	0.0398	0.0787	0.1166	0.1536	0.1238	0.1485	0.1980
EF	0.0493	0.0505	0.0517	0.0529	0.1090	0.1090	0.1090
EU	1.0240	1.0240	1.0240	1.0240	1.0244	1.0244	1.0244

Panel B: $\rho = -0.3$

$y = a_1 + a_2$	0.25	0.50	0.75	1.00	1.25	1.50	2.00
a_1	0.1250	0.2500	0.3750	0.5000	0.6250	0.7500	1.0000
a_2	0.1250	0.2500	0.3750	0.5000	0.6250	0.7500	1.0000
b_1	0.0123	0.0244	0.0361	0.0476	0.0000	0.0000	0.0000
b_2	0.0000	0.0000	0.0000	0.0000	3.0388	1.8233	1.2155
x	7.2322	3.6607	2.4702	1.8750	2.6252	2.1877	1.6408
ER	1.0123	1.0244	1.0361	1.0476	1.0357	1.0429	1.0572
$\sigma(r)$	0.0292	0.0577	0.0855	0.1127	0.0825	0.0990	0.1320
EF	0.0915	0.0937	0.0960	0.0982	0.2343	0.2343	0.2343
EU	1.0446	1.0446	1.0446	1.0446	1.0469	1.0469	1.0469

Table 7: Optimal Contracts in the Fulcrum Fee case with Fund Manager Effort

This table presents the equilibrium values in a fulcrum fee regime of various quantities for given values of the fund's portfolio choice a: (i) the fee structure optimal values of b_1 and b_2, (ii) optimal effort (e), (iii) the amount x invested in the fund, (iv) the net-of-fees expected return ER to the investor, (v) the volatility of these returns $\sigma(R)$, (vi) the fund's expected fees EF, (vii) the investor's expected utility EU. The cost function was assumed to be of quadratic form, i.e. ke^2, where $k = 5$. Searching over risk level provides the optimal fund type and compensation scheme. The parameters chosen were as follows: investor risk aversion ($\gamma = 2$), benchmark return parameters ($\pi_1 = 0.15$, $\pi_2 = 0.05$).

Cost of Effort $k = 5$

Parameters	Risk Level (a)						
	0.25	0.5	0.75	1.00	1.25	1.5	2
b_1	0.0042	0.0084	0.0125	0.0166	0.0184	0.0219	0.0288
b_2	0.0000	0.0000	0.0000	0.0000	0.5817	0.3575	0.2495
e	0.0054	0.0054	0.0055	0.0055	0.0079	0.0080	0.0082
x	4.6538	2.3367	1.5644	1.1782	1.0726	0.9005	0.6855
ER	1.0042	1.0084	1.0125	1.0166	1.0183	1.0218	1.0287
$\sigma(r)$	0.0212	0.0423	0.0632	0.0839	0.0924	0.1101	0.1447
EF	0.0195	0.0197	0.0199	0.0200	0.0254	0.0257	0.0264
EU	1.0098	1.0098	1.0098	1.0098	1.0099	1.0099	1.0099

Cost of Effort $k = 10$

Parameters	Risk Level (a)						
	0.25	0.5	0.75	1.00	1.25	1.5	2
b_1	0.0042	0.0083	0.0124	0.0165	0.0182	0.0217	0.0285
b_2	0.0000	0.0000	0.0000	0.0000	0.5770	0.3546	0.2474
e	0.0027	0.0027	0.0027	0.0028	0.0039	0.0040	0.0041
x	4.6442	2.3318	1.5611	1.1757	1.0684	0.8969	0.6825
ER	1.0042	1.0083	1.0124	1.0165	1.0182	1.0217	1.0285
$\sigma(r)$	0.0212	0.0422	0.0631	0.0837	0.0923	0.1099	0.1445
EF	0.0195	0.0196	0.0198	0.0199	0.0253	0.0256	0.0262
EU	1.0097	1.0097	1.0097	1.0097	1.0097	1.0097	1.0097

Table 8: Optimal Contracts in the Performance Fee case with Fund Manager Effort

This table presents the equilibrium values in an incentive-fee regime of various quantities for given values of the fund's portfolio choice a: (i) the fee structure optimal values of b_1 and b_2, (ii) optimal effort (e), (iii) the amount x invested in the fund, (iv) the net-of-fees expected return ER to the investor, (v) the volatility of these returns $\sigma(R)$, (vi) the fund's expected fees EF, (vii) the investor's expected utility EU. The cost function was assumed to be of quadratic form, i.e. ke^2, where $k = 5$. Searching over risk level provides the optimal fund type and compensation scheme. The parameters chosen were as follows: investor risk aversion ($\gamma = 2$), benchmark return parameters ($\pi_1 = 0.15$, $\pi_2 = 0.05$).

Cost of Effort $k = 5$

Parameters	Risk Level (a)						
	0.25	0.5	0.75	1.00	1.25	1.5	2
b_1	0.0042	0.0084	0.0125	0.0166	0.0000	0.0000	0.0000
b_2	0.0000	0.0000	0.0000	0.0000	2.3501	1.4090	0.9380
e	0.0054	0.0054	0.0055	0.0055	0.0184	0.0183	0.0180
x	4.6538	2.3367	1.5644	1.1782	1.5022	1.2515	0.9382
ER	1.0042	1.0084	1.0125	1.0166	1.0137	1.0165	1.0220
$\sigma(r)$	0.0212	0.0423	0.0632	0.0839	0.0676	0.0811	0.1082
EF	0.0195	0.0197	0.0199	0.0200	0.0424	0.0424	0.0424
EU	1.0098	1.0098	1.0098	1.0098	1.0103	1.0103	1.0103

Cost of Effort $k = 10$

Parameters	Risk Level (a)						
	0.25	0.5	0.75	1.00	1.25	1.5	2
b_1	0.0042	0.0083	0.0124	0.0165	0.0000	0.0000	0.0000
b_2	0.0000	0.0000	0.0000	0.00007	2.2995	1.3792	0.9189
e	0.0027	0.0027	0.0027	0.0028	0.0090	0.0089	0.0088
x	4.6442	2.3318	1.5611	1.1757	1.4758	1.2297	0.9221
ER	1.0042	1.0083	1.0124	1.0165	1.0136	1.0164	1.0218
$\sigma(r)$	0.0212	0.0422	0.0631	0.0837	0.0680	0.0815	0.1087
EF	0.0195	0.0196	0.0198	0.0200	0.0416	0.0416	0.0416
EU	1.0097	1.0097	1.0097	1.0097	1.0101	1.0101	1.0101

Table 9: Comparison of Equilibria at various cost and effort levels

This table presents differences in three measures of welfare between the two fee structures (fulcrum and performance fees). These are summarized values from Tables 1,2,7 and 8. The differences are tabulated for three cost levels, i.e. $k = 5, k = 10$ and $k = \infty$. The three measures are the standard deviation of net returns, the expected fees after effort costs, expected utility of net returns. Values in the table are the value of the welfare measure in the performance fee model minus that of the fulcrum fee model. Since these differ only when the fund is riskier than its benchmark, we present values only when $a > 1$.

Cost of effort $k = 5$				
Differences in	$a = 1.01$	$a = 1.25$	$a = 1.50$	$a = 2.00$
$\sigma(r)$	-0.0206	-0.0248	-0.0289	-0.0365
EF	0.0174	0.0170	0.0167	0.0160
EU	0.0005	0.0005	0.0005	0.0005
Cost of effort $k = 10$				
Differences in	$a = 1.01$	$a = 1.25$	$a = 1.50$	$a = 2.00$
$\sigma(r)$	-0.0202	-0.0243	-0.0284	-0.0358
EF	0.0167	0.0163	0.0160	0.0154
EU	0.0004	0.0004	0.0004	0.0004
Cost of effort $k = \infty$				
Differences in	$a = 1.01$	$a = 1.25$	$a = 1.50$	$a = 2.00$
$\sigma(r)$	-0.0198	-0.0239	-0.0279	-0.0351
EF	0.0160	0.0157	0.0154	0.0147
EU	0.0002	0.0002	0.0002	0.0002

Table 10: Equilibria when the Fund is also Risk-Averse

This table presents equilibrium outcomes for the model of Section 4 when the fund's utility function is given by $u(x) = x^{\gamma}$. Two values are considered for γ, and three values for the choice of fund portfolio a. The remaining parameters are fixed as in Table 1. In each case, equilibrium values are given for three quantities: the investor's equilibirum expected utility EU, the fund's equilibrium expected utility EF, and the volatility of net returns to the investor $\sigma(R)$.

$\gamma = 0.9$

Incentive Fee Outcomes

a	1.25	1.50	2.00
EU	1.0098	1.0098	1.0098
EF	0.0504	0.0504	0.0504
$\sigma(R)$	0.0683	0.0819	0.1092

Fulcrum Fee Outcomes

a	1.25	1.50	2.00
EU	1.0096	1.0096	1.0096
EF	0.0357	0.0361	0.0368
$\sigma(R)$	0.0922	0.1098	0.1443

$\gamma = 0.5$

Incentive Fee Outcomes

a	1.25	1.50	2.00
EU	1.0097	1.0097	1.0097
EF	0.1546	0.1546	0.1547
$\sigma(R)$	0.0829	0.0995	0.1327

Fulcrum Fee Outcomes

a	1.25	1.50	2.00
EU	1.0096	1.0096	1.0096
EF	0.1515	0.1518	0.1523
$\sigma(R)$	0.0922	0.1098	0.1443

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